AUTOMATIC FLIGHT CONTROL SYSTEMS

3. ANALYSIS OF FEEDBACK CONTROL SYSTEMS

TEST INPUT SIGNAL:

Test input signal is a technique of testing the stability of control systems by following given equations. There are four types of test functions for testing the input signal.

**STEP FUNCTION:**

\[ a(t) = \begin{cases} 0 & \text{when } (t \leq 0) \\ 1 & \text{when } (0 \leq t) \end{cases} \]

taking Laplace of a \( f(t) \) signal

\[ L f(t) = \int_{0}^{\infty} f(t) e^{-st} \, dt \]

**RAMP FUNCTION:**

\[ r(t) = \begin{cases} 0 & \text{when } t < 0 \\ k t & \text{when } t \geq 0 \end{cases} \]

taking Laplace

\[ L r(t) = \int_{0}^{\infty} r(t) e^{-st} \, dt \]

\[ R(s) = \frac{k}{s} \quad \text{(for unit ramp } k=1) \]

\[ R(s) = \frac{k}{s^2} \]
PARABOLIC FUNCTION:
\[ r(t) = \begin{cases} 0 & \text{when } t < 0 \\ \frac{y^2}{2} & \text{when } t > 0 \end{cases} \]
for unit parabolic \(k=1\)
\[ r(t) = \frac{y^2}{2} \quad t > 0 \]

Laplace of function
\[ L[r(t)] = \int_0^\infty r(t)e^{-st}\,dt = \frac{2}{s^3} \]
\[ R(s) = \frac{2}{s^3} \]

IMPULSE FUNCTION:
\[ \delta(t) = \begin{cases} \infty & \text{when } t \neq 0 \\ \lambda & \text{when } t = 0 \end{cases} \]
\[ \int_{-\infty}^{\infty} \delta(t)\,dt = \text{after integration} \]
\[ \delta(t) = \delta(t) \]

The pulse for which the duration tends to zero and amplitude tend to infinity is called impulse function.

TIME DOMAIN PERFORMANCE OF CHARACTERISTICS OF FEEDBACK CONTROL SYSTEMS:

DAMPING RATIO:
\[ \frac{\omega_n}{\zeta} = \frac{\omega_n}{\zeta} \]
Underdamped case \((0 < \zeta < 1)\)
Undamped case \((\zeta = 0)\)
Critically damped case \((\zeta = 1)\)
EFFECT OF DERIVATIVE AND INTEGRAL CONTROL:

Derivative control action:

\[
\frac{\text{?} (\text{?})}{\text{?} (\text{?})} = K_d
\]

\(K_d\) is the derivative constant. \(E(S)\) is the input signal and \(M(s)\) is the output signal.

Integral control action:

\[
\frac{\text{?} (\text{?})}{\text{?} (\text{?})} = \frac{\text{?} (\text{?})}{\text{?} (\text{?})}
\]

\(K_i\) is the integral constant.

Derivative and integral control functions increases the sensitivity of control systems.

STEADY STATE ERROR:

Static position error coefficient:

\[
\frac{\text{?} (\text{?})}{\text{?} (\text{?})} = \frac{1}{1+\text{?} (\text{?})}
\]

where \(k_p\) is the static position coefficient constant

Static velocity error coefficient:

\[
\frac{\text{?} (\text{?})}{\text{?} (\text{?})} = \frac{1}{\text{?} (\text{?})}
\]

where \(k_v\) is the velocity error coefficient constant

Static acceleration error coefficient:

\[
\frac{\text{?} (\text{?})}{\text{?} (\text{?})} = \frac{1}{\text{?} (\text{?})}
\]

where \(k_a\) is the acceleration error coefficient constant

By Anand kumar Jha, student of aeronautical engineering