

2. FEED BACK CONTROL SYSTEM

-TRANSFER FUNCTION OF LINEAR SYSTEMS

Linear engineering systems are those that can be modelled by linear differential equations. We shall only consider those systems that can be modelled by constant coefficient ordinary differential equations.

Consider a system modelled by the second order differential equations

$$a \frac{d^2y}{dt^2} + b \frac{dy}{dt} + cy = f(t) \quad : \text{ } ^2 \text{ is power of square}$$

where $f(t)$ is the input signal and the solution of this equation is $y(t)$, a, b, c are the constants.

Laplace transformation of this equation is :-

$$(as^2 + bs + c)Y(s) = F(s)$$

In which we have used $y(0) = y'(0) = 0$ and where we have designated $L\{y(t)\} = Y(s)$ and $L\{f(t)\} = F(s)$.

We define the transfer function of a system to be the ratio of the laplace transform of the output signal to the input signal with the initial conditions as zero. The transfer function (a function of s), is denoted by $H(s)$.

$$H(s) = \frac{Y(s)}{F(s)} = \frac{1}{as^2 + bs + c}$$

IMPULSE RESPONSE OF THE LINEAR SYSTEMS:

Now in the special case in which the input signal is the delta function $f(t) = \delta(t)$ we have $F(s) = 1$

$$H(s) = Y(s)$$

The differential equation in this special case the unit impulse response function and denote it by $h(t)u(t)$.

$$h(t)u(t) = L^{-1}\{H(s)\} \quad \text{when } f(t) = \delta(t)$$

$\delta(t)$ is the impulse function

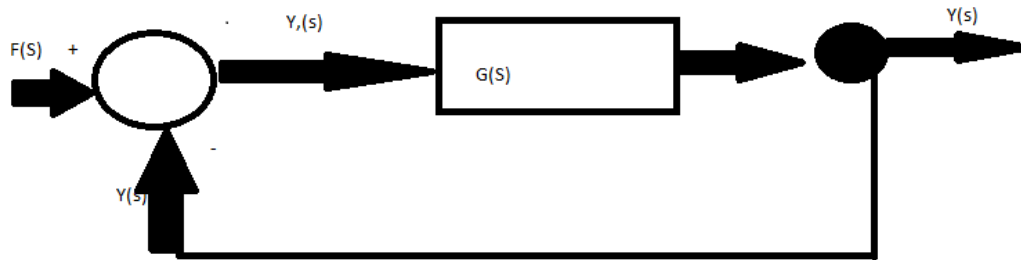
$$Y(s) = H(s)F(s)$$

Output signal is as usual obtained by taking the inverse laplace transform:

$$Y(t) = L^{-1}\{Y(s)\} = \frac{L^{-1}\{H(s)F(s)\}}{h(t)f(t)}$$

$Y(t)$ is the convolution. The solution to a linear system modelled by the transformation coefficient ordinary differential equation is given by the convolution of the unit in pulse response function $h(t)u(t)$ with the input function $f(t)$.

BLOCK DIAGRAM OF FEED BACK CONTROL SYSTEMS:



BLOCK DIAGRAM OF THE FEED BACK CONTROL SYSTEM

Here the output signal is tapped and subtracted from the input signal. Hence

$$Y(s) = G(s)Y_1(s)$$

Because $Y_1(s)$ is the input signal to the system characterised by transfer function $G(s)$. However at the summing point $Y_1(s) = F(s) - Y(s)$

$$Y(s) = G(s)\{F(s) - Y(s)\}$$

$$Y(s) = \frac{G(s)F(s)}{1 + G(s)}$$

In terms of input and output signal the feedback loop is characterised by transfer function _____

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