

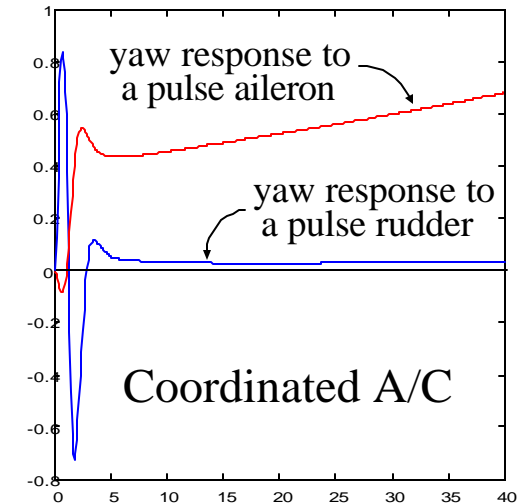
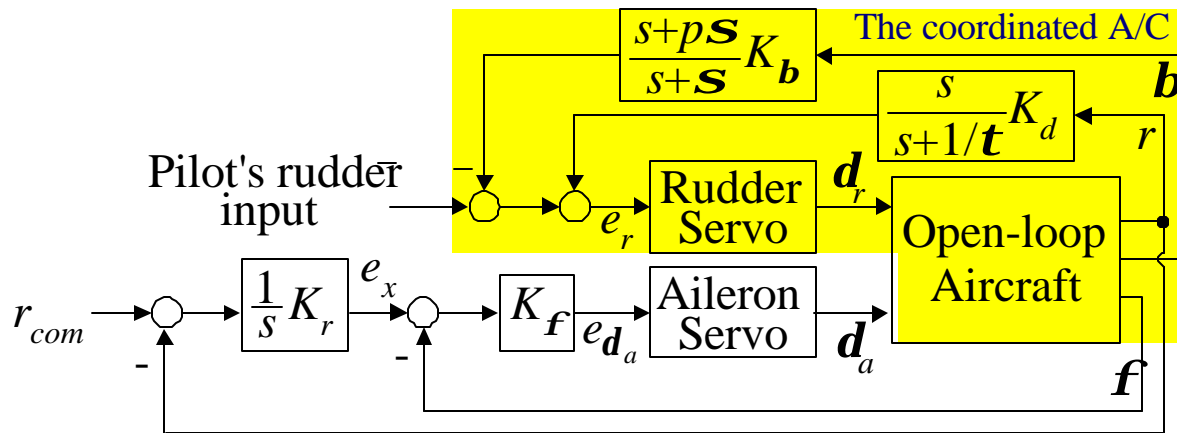
《Intermediate Lateral Autopilots (I) – Yaw orientation control》

Yaw orientation autopilot – Lateral autopilot for yaw maneuver

Designed to have the aircraft follow the pilot's yaw rate command or hold the aircraft with a reference yaw rate signal. ==> The autopilot will work on the *coordinated aircraft*

--- A *coordinated A/C* will mean A/C with the Dutch roll damper and the coordination controller.

Typical block diagram:

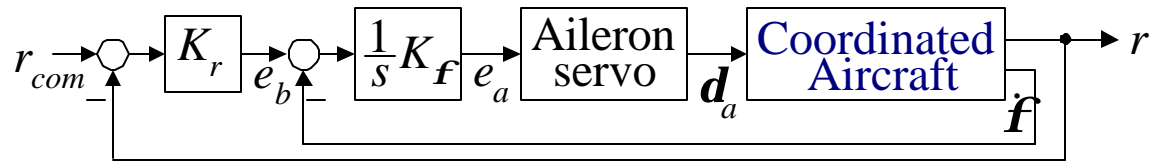


Important features of the design:

- The controlling input turns to the aileron, instead of the rudder.
 - From the response plot shown in the right figure above, we can see that, *in a coordinated A/C, the aileron becomes a more effective input for yaw motion.* (Appendix B)
- An integrator feedback is included to remove the steady state error between $r(t)$ and r_{com}
 - The coordinated A/C is normally with an unstable spiral mode, and the integrator feedback will further destabilize the system
- An additional f feedback inner-loop is therefore included to stabilize the lateral motion.
 - A roll angle feedback is most effective for stabilizing the spiral mode.

Working block diagram

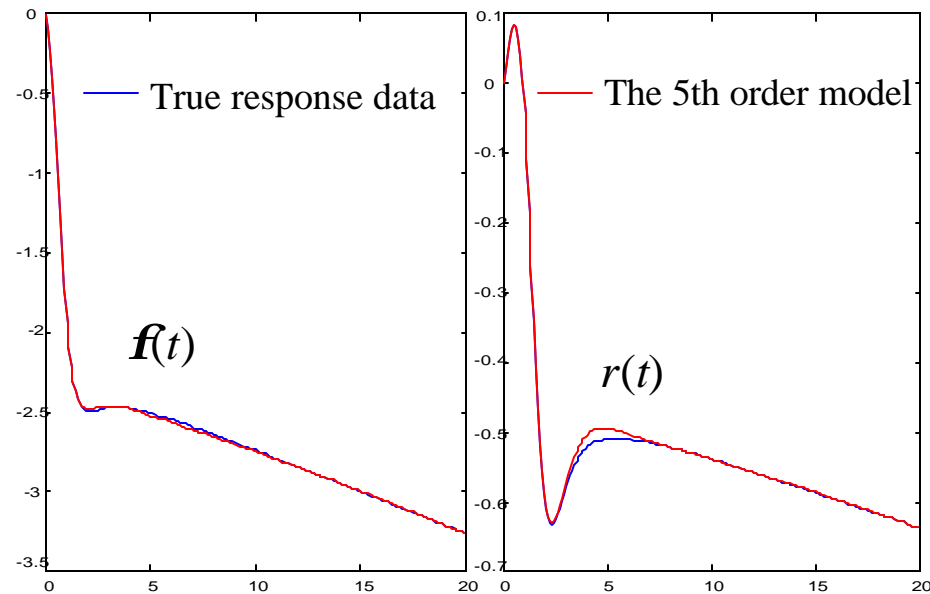
- *The block diagram shows what is implemented.*



- The roll angle feedback is replaced with a roll rate feedback.
 - Roll rate signal is much easier to obtain than the roll angle signal.
 - As the turn gets steady, $\dot{f} = 0$ and the roll rate feedback terminates (self-washout).

A/C model for the design:

- Require d_a to f and d_a to r transfer functions of the coordinated A/C.
- Approximated model estimated from the response data (of the coordinated A/C) will be used.
 - True model of the coordinated A/C is complicated to compute (see Appendix C).
- The following fifth order model was estimated for the coordinated A/C presented in p.84 of this note:

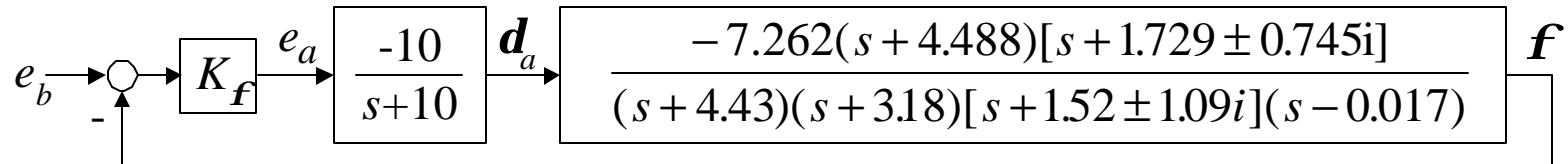


$$r(s) = \frac{1.76(s + 8.406)(s + 0.767)(s - 1.967)}{(s + 4.427)(s + 3.179)[s + 1.516 \pm 1.086i](s - 0.017)} d_a(s)$$

$$f(s) = \frac{-7.262(s + 4.488)[s + 1.729 \pm 0.745i]}{(s + 4.427)(s + 3.179)[s + 1.516 \pm 1.086i](s - 0.017)} d_a(s)$$

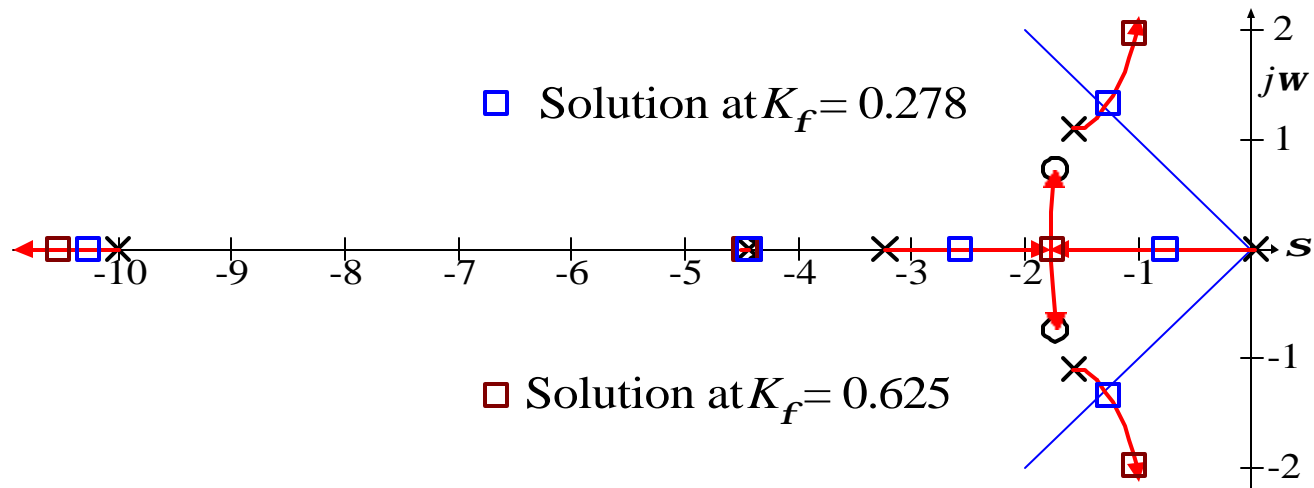
Inner-loop analysis

○ Inner-loop block diagram:



--- We have retained the roll angle feedback format, to simplify the analysis.

--- A negative gain aileron servo is used, because that $\mathbf{f}(s) / \mathbf{d}_a(s)$ has a negative gain.



--- The spiral mode will be stabilized, but the Dutch roll mode will suffer, by the inner-loop feedback. However, the later will regain its nice damping with the outer-loop feedback.

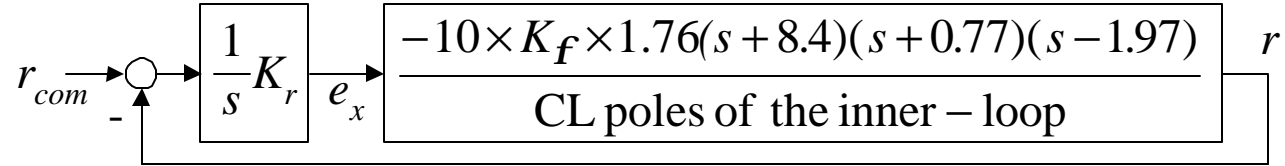
--- $K_f = 0.625$ corresponds to the highest stability for the spiral mode.

--- However, a zero at $s = -0.767$ will appear in the outer-loop locus. If not removed, this zero will stop the outer-loop integrator pole from going left.

--- $K_f = 0.278$ will produce a inner-loop CL pole to cancel the outer-loop zero at $s = -0.767$

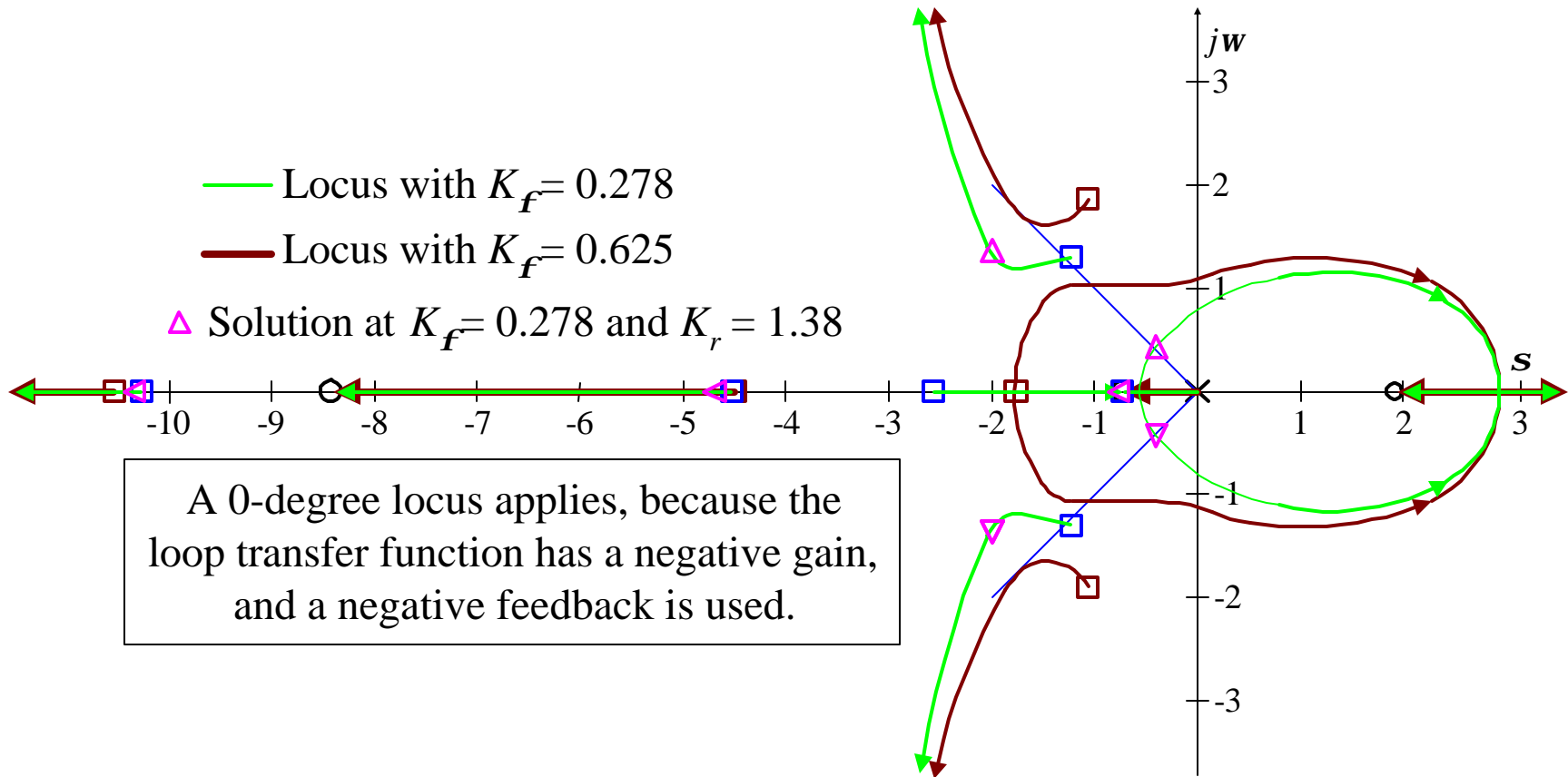
Outer-loop analysis:

- The numerator of the loop transfer function has been



changed from the numerator of $f(s)/d_a(s)$ to the numerator of $r(s)/d_a(s)$.

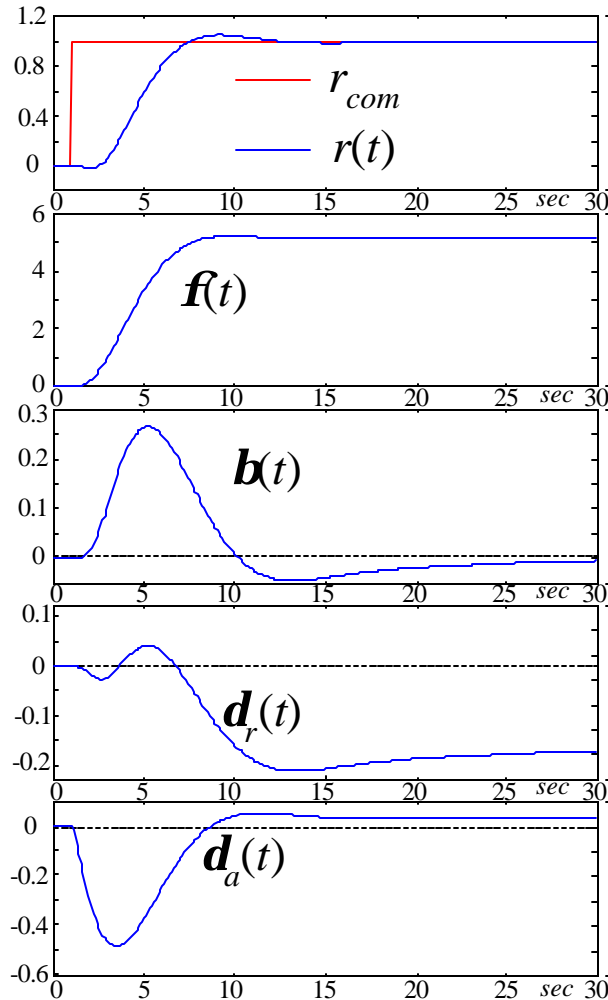
--- Because that the two feedback loops are for the same controlling input, change in feedback signal is equivalent to change in the numerator of the corresponding model.



Closed-loop simulation of the yaw orientation autopilot:

○ The simulated design was with $K_f = 0.278$ and $K_r = 1.38$.

--- For the coordinated A/C, we have used the example presented on p.84 of this note.



$r(t)$ does follow r_{com} , though with certain time delay, a result of the high relative order of the system

A steady state $f(t)$ is also established, creating a steady turn.

A coordinated turn is established, the steady state sideslip is nearly nulled..

A steady state $d_r(t)$ results to hold the sustained yaw rate

The steady state $d_a(t)$ is to compensate the unbalanced relative wind on two sides of the wing due to yaw

--- Basically, all lateral controllers discussed thus far perform as they are designed to.

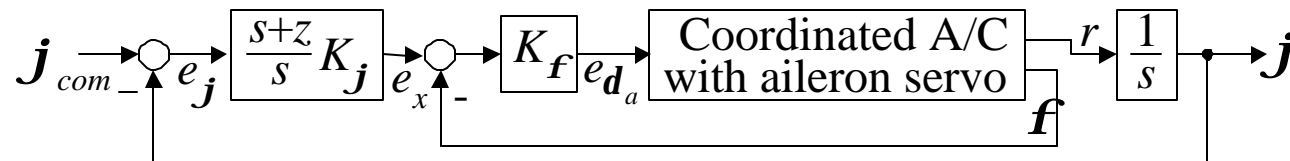
《Intermediate Lateral Autopilots (II) – Heading Autopilot pilot》

Heading autopilot: A displacement autopilot for yaw

Preliminaries about the autopilot:

- Design goal: To have the aircraft follow a reference heading signal Ψ_{com}
 - In general, Ψ is the heading of the A/C in the horizontal plane.
 - For small bank angle, $\mathbf{f} \ll 1$, we will have $r = \dot{\Psi} \cos \mathbf{f} \approx \dot{\Psi}$; hence, $\Psi \approx r/s = \mathbf{j}$
 - As a result, we can treat Ψ_{com} as a yaw angle command \mathbf{j}_{com} .
 - $\mathbf{j}_{com}(\Psi_{com})$ signal may be generated through integrating r_{com} in a pilot operated maneuver or be sensed by a directional gyro in an automatic flight control loop.
- Again, this autopilot will work on the coordinated aircraft

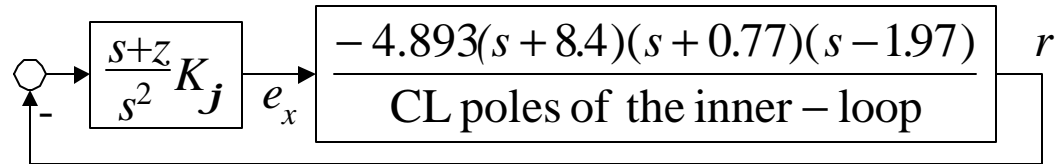
Typical block diagram:



- A roll angle feedback inner-loop is kept here for stabilization.
- An integrator is also adopted to ensure steady state command following.
- A zero at $s = -z, z > 0$ is included to attract the locus to enter the LHP.
 - The \mathbf{j} -feedback introduces a pole at $s = 0$ to the system. With an integrator control, the outer-loop locus from the double integrator will not enter the LHP without a zero nearby.
- The inner-loop portion of this design is the same as that of the yaw orientation autopilot.
- We will go right with the outer-loop locus analysis. We will also set $K_f = 0.278$.

Outer-loop root locus:

- Please refer this block diagram to that of the yaw orientation autopilot design.

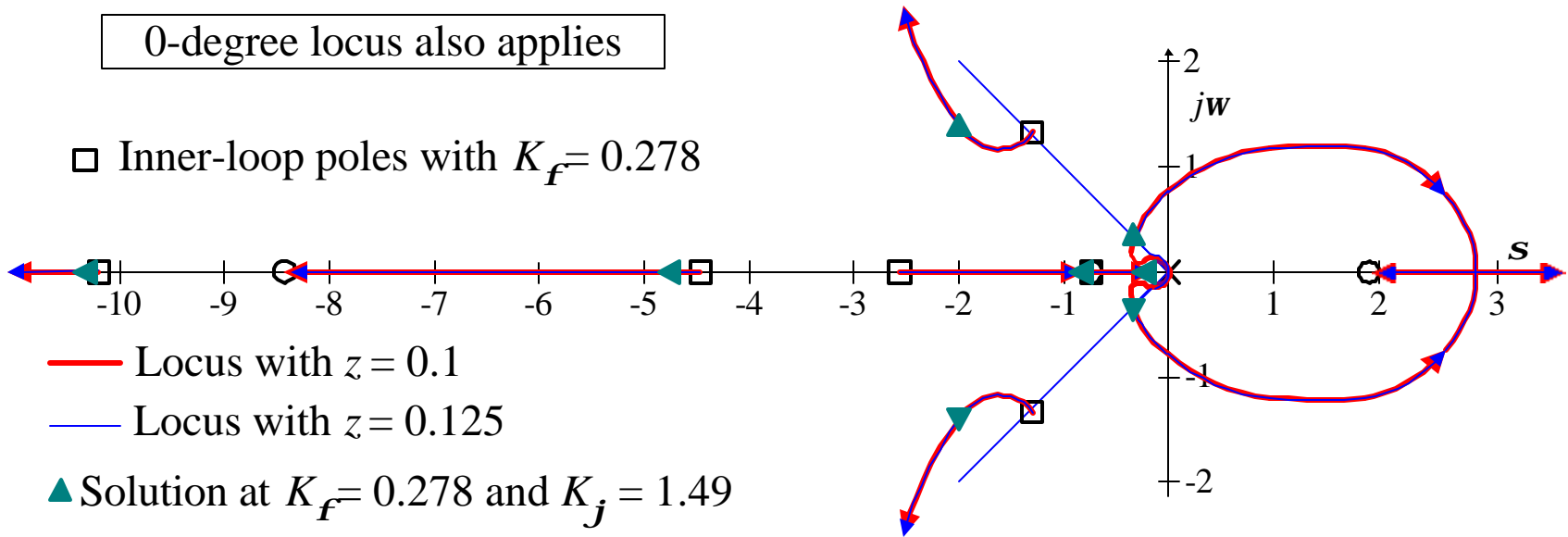


- *Note that this block diagram is for root locus analysis only.* For the real system, its output is the yaw angle \mathbf{j} .

The outer-loop locus

0-degree locus also applies

- Inner-loop poles with $K_f = 0.278$



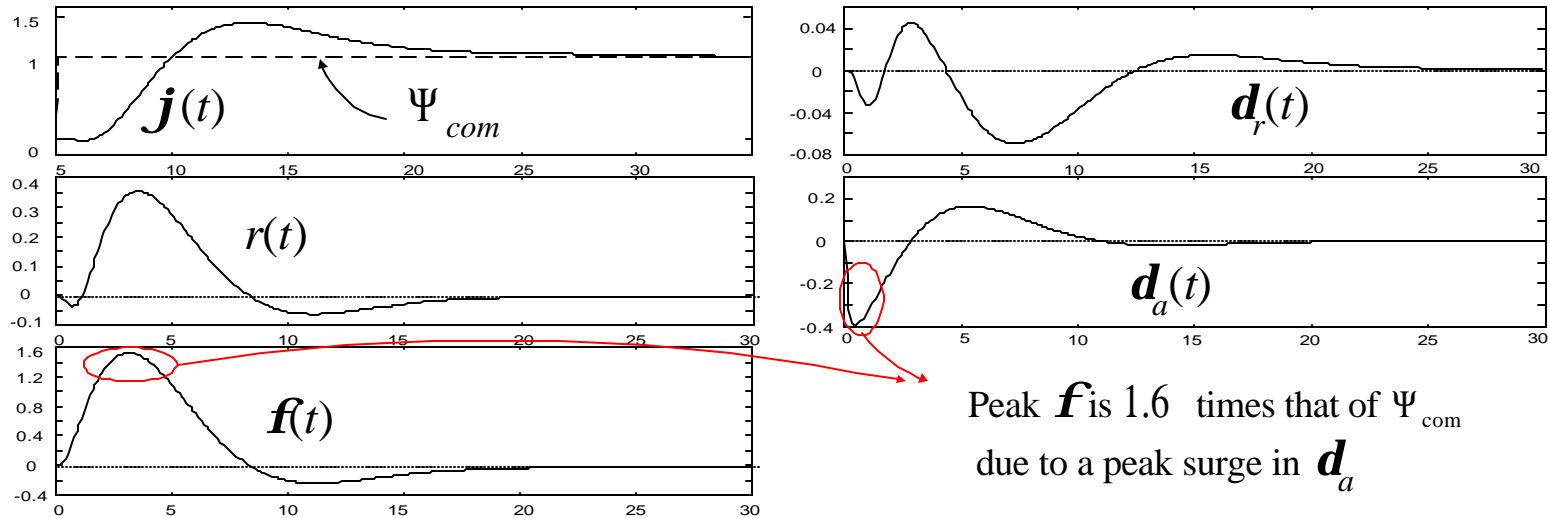
— Locus with $z = 0.1$

— Locus with $z = 0.125$

▲ Solution at $K_f = 0.278$ and $K_j = 1.49$

- Two complex pole pairs result.
 - The Dutch roll pair will increase in stability, and is of no concern here.
 - The dominant pair will lose its obtainable damping ratio with increasing value of z .
 - If a damping ratio of 0.707 is a specification, then $z = 0.125$ is the upper bound for z .
- The zero at $s = -z$ will also attract a CL pole near by. Hence, a smaller z is not desirable,
 - A trade off design is necessary. Final choice may be determined from a CL simulation.

Closed-loop system response:

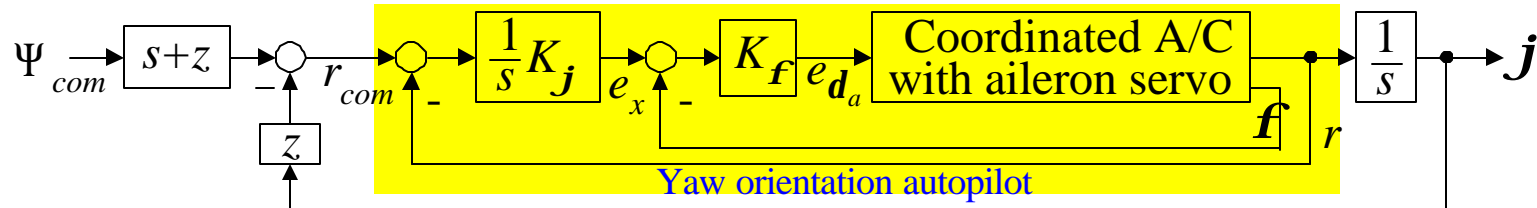


- This simulation is performed with $z = 0.1$ and $K_b = 1.49$, and on a coordinated A/C.
- The heading error of the A/C is forced to zero at the steady state.
- In addition, because that $\lim_{t \rightarrow \infty} f(t) = 0$, the assumption, $\mathbf{j}_{com} \approx \Psi_{com}$, or in a more general sense, $\mathbf{j} \approx \Psi$ for any \mathbf{j} , is valid.
 - This result will allow us to control the A/C heading through control of the yaw angle \mathbf{j} .
- However, a large peak value of the bank angle occurred during the maneuver.
 - With a peak f that equals $1.6\Psi_{com}$, a 30° turn maneuver would produce a peak f of 50°
 - Normally, $30^\circ \approx 0.5 \text{ rad}$ will be the limit for f before the passengers begin to feel uncomfortable, or even panic, about the flight.
 - The large peak value in f is a result of the large yaw rate that occurs during the maneuver.
 - Some form of the improvement on the heading pilot design is necessary.

Improved design: Heading autopilot with a yaw limiter

Remedy to the peak surge in bank angle:

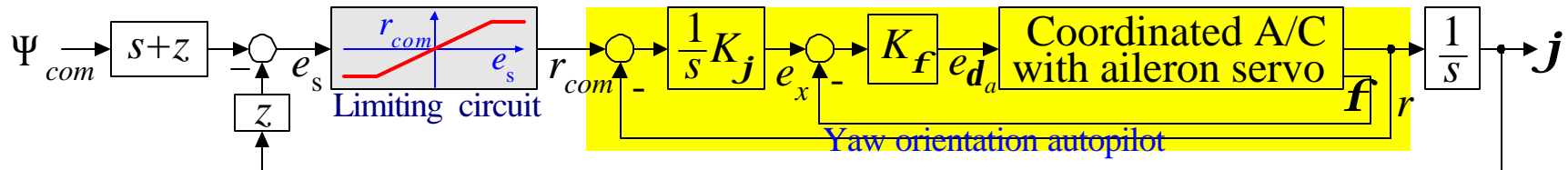
- First of all, the following equivalent form of the heading autopilot can be drawn:



==> A heading autopilot is a heading feedback outer-loop to a yaw orientation autopilot.

--- Implementing the heading autopilot in this form will require differentiation on the heading command Ψ_{com} . However, any noise amplification due to this differentiation will be restored by the integrator inside the second feedback loop.

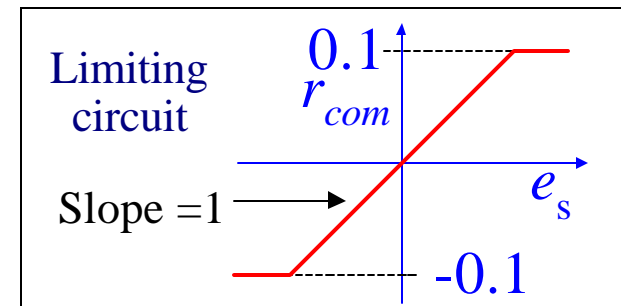
- Since we regard the large peak value in f is a result of the large yaw rate that occurs during the maneuver., we can improve this effect by limiting the yaw rate as follows:



--- A saturation limit on r_{com} is installed.

Design of the yaw rate limiting circuit:

- Assume that a $f \leq 30^\circ \approx 0.5 \text{ rad}$ is sought.
- CL simulation of the yaw orientation autopilot reveals that the ratio between f and r will be about 5:1.
- Then, a limit of r_{com} can be set at $r_{com} \leq 0.1 \text{ rad/sec}$.



《Intermediate Lateral Autopilots (III) – Roll orientation control》

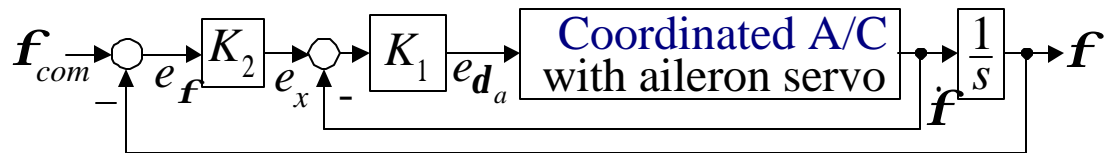
Roll orientation autopilot – A command following control for roll

Definition of the problem:

- Design an autopilot to have the A/C follows the pilot's roll angle command or hold the aircraft with a reference roll angle.
- The autopilot will also work on the **coordinated aircraft**

Typical block diagram

- A roll rate feedback inner-loop is kept for stability improvement.

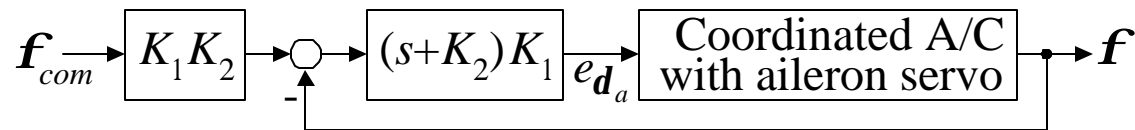


- The feedback structure is equivalent to a proportional plus differentiator (PD) control:

--- Mathematically,
$$e_{d_a}(s) = K_1[K_2(\mathbf{f}_{com}(s) - \mathbf{f}(s)) - \dot{\mathbf{f}}(s)]$$

$$= K_1K_2\mathbf{f}_{com}(s) - K_1(s + K_2)\mathbf{f}(s)$$

- Equivalent block diagram:



==> The PD control structure seems indicate that steady state output error may persist.

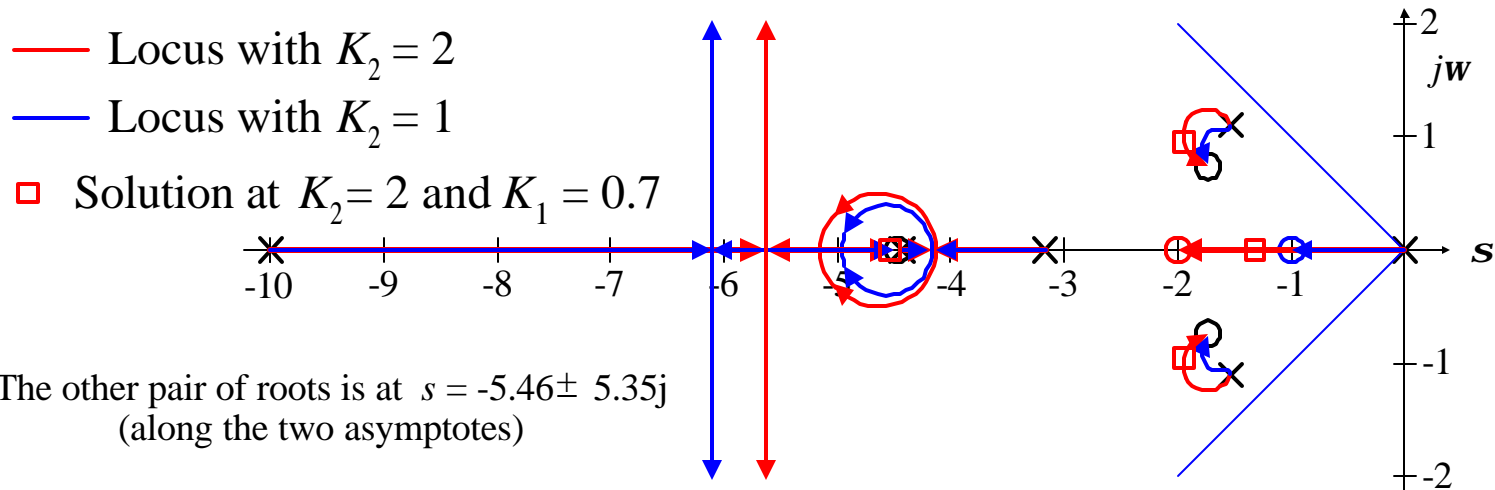
Dynamic model of the coordinated A/C and of the aileron servo:

- For the same coordinated aircraft used in the yaw orientation autopilot design, we have:

$$\frac{\mathbf{f}(s)}{\mathbf{d}_a(s)} = \frac{-7.262(s + 4.488)[s + 1.729 \pm 0.745i]}{(s + 4.427)(s + 3.179)[s + 1.516 \pm 1.086i](s - 0.017)}, \quad \frac{\mathbf{d}_a(s)}{e_{d_a}(s)} = \frac{-10}{s + 10}$$

- A positive gain will result for the overall system; hence, a 180-degree locus will apply.

Closed-loop system of the design



The other pair of roots is at $s = -5.46 \pm 5.35j$
(along the two asymptotes)

- The leftward moment of the spiral model will increase with larger value of K_2 .
- The following CL system is also obtained for the selected controller gains:

$$TF_{CL}(s) = \frac{\mathbf{f}(s)}{\mathbf{f}_{com}(s)} = \frac{50.8(s+2)(s+4.488)[s+1.729 \pm 0.745i]}{(s+1.307)(s+4.516)[s+1.936 \pm 0.958i][s+5.46 \pm 5.347i]}$$

$$= \frac{50.8s^4 + 505.5s^3 + 1776.6s^2 + 2745.9s + 1616.6}{s^6 + 20.6s^5 + 197.5s^4 + 978.2s^3 + 2508.5s^2 + 3222.6s + 1608.2}$$

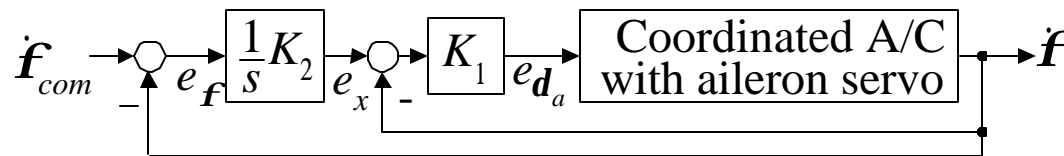
- A dominant CL pole at $s = -1.3$ results, indicating a fast CL response (in about 3 sec).
- Also, with $TF_{CL}(s)|_{s=0} = 1.005$, a 0.5% steady state error also results for a step $\mathbf{f}_{com}(s)$
- For a PD feedback system, this near perfect output following is unusual.
- The fact is that the spiral mode of the open-loop model, being so close to the origin, acts like an integrator, thereby squeezing out the steady state output error.

Roll orientation autopilot – A rate command version

Address the problem:

- A roll angle command is used in the previous design.
- In a manual maneuver, the pilot's control stick input normally represents a roll rate command.
- Need to provide the pilot with a capability to maneuver the A/C with a roll rate command.

Block diagram of a roll orientation autopilot with a roll rate command:



--- A roll rate feedback inner-loop is still kept for stability improvement.

--- An integrator is also included to ensure the rate command following.

- The CL system of this design is exactly the same as that of the previous design.

--- For the same A/C and the same controller gains, the CL system will remain as

$$\frac{\dot{f}(s)}{\dot{f}_{com}(s)} = \frac{50.8s^4 + 505.5s^3 + 1776.6s^2 + 2745.9s + 1616.6}{s^6 + 20.6s^5 + 197.5s^4 + 978.2s^3 + 2508.5s^2 + 3222.6s + 1608.2}$$

--- Again, a nice steady state command following will results for a step $\dot{f}_{com}(s)$

Closed-loop system response to a pulse roll rate command:

